

Math Virtual Learning

# AP Statistics

## 2 Prop z-tests

April 23rd, 2020



Lesson: April 23rd, 2020

**Objective/Learning Target:**

Students will be able to apply the 2 proportion z-test and confidence interval methods to solve problems.

# Review #1

The P-value of a test of a null hypothesis is the probability that...

- A) assuming the null hypothesis is true, the test statistic will take a value at least as extreme as that actually observed.
- B) assuming the null hypothesis is false, the test statistic will take a value at least as extreme as that actually observed.
- C) the null hypothesis is true.
- D) the null hypothesis is false.
- E) the alternative hypothesis is true.

## Review #2

In a statistical test of significance, we say the data are statistically significant at level  $\alpha$  if

A)  $\alpha = 0.01$ .

B)  $\alpha = 0.05$ .

C) the P-value is at most  $\alpha$ .

D) the P-value is larger than  $\alpha$ .

E)  $\alpha$  is small.

# Answers

Review #1: The answer is A, we are always assuming the null hypothesis is true until we have evidence that it is not. The p-value is the area under the curve created by the null hypothesis that is at least as far from the null value as the sample gives. We never actually have a probability the null or the alternative is true.

Review #2: The answer is C. We are always looking for the p-value to be less than the alpha value. A p-value larger than alpha causes us to fail to reject the null. We can choose the level of alpha based on how willing we are to have type 1 error.


# $p_1-p_2$ Sampling distribution

Recall the sampling distribution of the difference of 2 proportions:

Center: It will have a mean p-hat of:

$$\mu_{p_1-p_2} = \mu_{p_1} - \mu_{p_2} = P_1 - P_2$$

Spread: The standard deviation of the distribution is:

$$\sigma_{p_1-p_2} = \sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}$$


For the test we will assume the null is true and use the same  $P$  for both samples. This  $P$  will be:  $P_c = (x_1 + x_2) / (n_1 + n_2)$ . This is called the pooled probability

Normal: We know that when we combine two normal distributions the resulting distribution is also normal!

# Developing a Test

The  $p_1$ - $p_2$  sampling distribution allows us to reduce two sampling distributions into a single one. Once we have a single distribution, we can perform a test on the resulting statistics much like we performed tests on a single proportion.

We will still use the formula:

$$\text{Standardized test statistic: } \frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$$

To calculate a z-score, and we can still convert that to a p-value in the same way. The only difference is the values we put in for the statistic, parameter, and standard error.

# Assumptions of the test

These are going to look familiar!

Random: Both  $p_1$  and  $p_2$  are from separate random samples. Note that they cannot be values of a single matched pairs sample... we need a matched pairs test procedure for this.

Independent: The samples need to be independent of each other, individuals in each sample need to be independent, and we cannot sample more than 10% of the population

Normal: Each sample needs to meet the  $np$  and  $n(1-p)$  condition.



# Example

High levels of cholesterol in the blood are associated with higher risk of heart attacks. Will using a drug to lower blood cholesterol reduce heart attacks? The Helsinki Heart Study looked at this question. Middle-aged men were assigned at random to one of two treatments: 2051 men took the drug gemfibrozil to reduce their blood cholesterol levels, and a control group of 2030 men took a placebo. During the next five years, 56 men in the gemfibrozil group and 84 men in the placebo group had heart attacks.

# State

We want to test to see if the men taking the medication have a lower proportion of heart attacks than the men on the placebo.

$P_1$  : The true proportion of men having heart attacks that are on the medication.

$P_2$ : The true proportion of men having heart attacks that are on the placebo.

Hypothesis formatted how we read them:

$$H_0: P_1 = P_2$$

$$H_a: P_1 < P_2$$



Hypotheses formatted for testing:

$$H_0: P_1 - P_2 = 0$$

$$H_a: P_1 - P_2 < 0$$

# Plan

We will use a 2-prop z-test with an  $\alpha=0.05$

Random: Both the medication and placebo group were assigned randomly.

Independent: The chance of heart attack in each group and among each individual can safely be assumed to be independent of each other... One person having a heart attack does not affect others chances. (note... if we had multiple family members in the study this would not be independent. Why?)

Normal: Both groups meet the  $np$  and  $n(1-p)$  condition. Note that when given counts, we can easily tell the  $n_1p_1 = 56$ ,  $n_1(1-p_1)=2051-56$ ,  $n_2p_2 = 84$ ,  $n_1(1-p_1)=2030-84$ . Essentially as long as all counts are greater than 10 you are good. Try it yourself to confirm.

# Do

$$x_1 = 56$$

$$x_2 = 84$$

$$n_1 = 2051$$

$$n_2 = 2030$$

$$p_1 = 56/2051 \approx 0.0273$$

$$p_2 = 84/2030 \approx 0.0414$$

$$p_c = (56+84)/(2051+2030) = 0.0343$$

Note the value of  $p_c$ . This is the pooled probability. We are using this because our assumption is the  $p_1=p_2$ . If the two probabilities are actually the same,  $p_c$  is a better estimate by the law of large numbers!

Do

$$\mu_{p1-p2} = \mu_{p1} - \mu_{p2} = P_1 - P_2 = 0.0273 - 0.0414 = -0.0141$$

$$\sigma_{P_c} = \sqrt{\frac{P_c(1-P_c)}{n_1} + \frac{P_c(1-P_c)}{n_2}} = \sqrt{\frac{0.0343(1-0.0343)}{2051} + \frac{0.0343(1-0.0343)}{2030}} \approx 0.00570$$

$$z\text{-score} = \frac{-0.0141 - 0}{0.00570} = -2.47 \rightarrow \text{from z-table p-value} = 0.0068$$

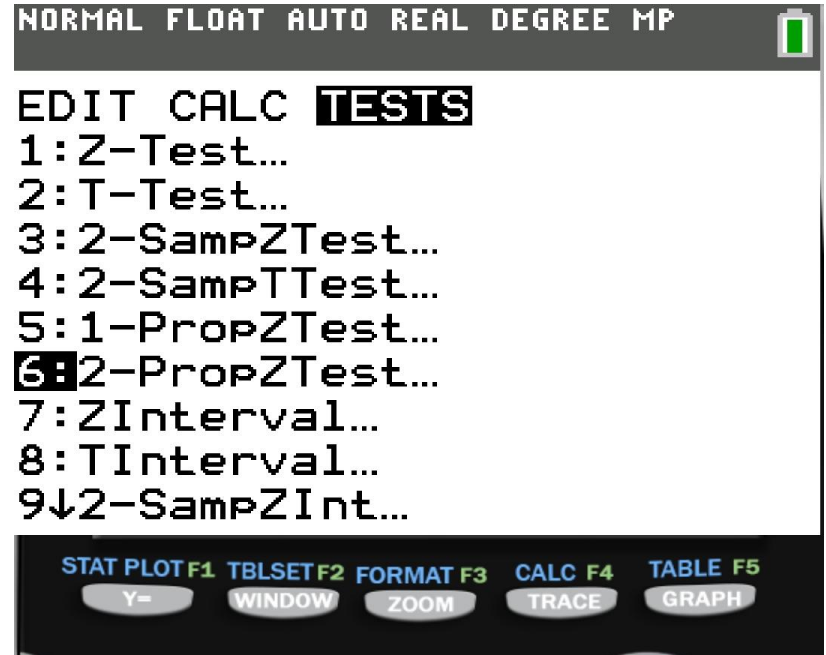
# Do - continued

A few notes about this step...

- If we had a two tailed test, we would simply double our p-value to account for the second tail.
- The parameter value of zero in the z-score equation is from our null hypothesis. Typically tests for the difference of two parameters will take a zero in the parameter position representing no difference between the two.
- This can be completed on the calculator, shown next.

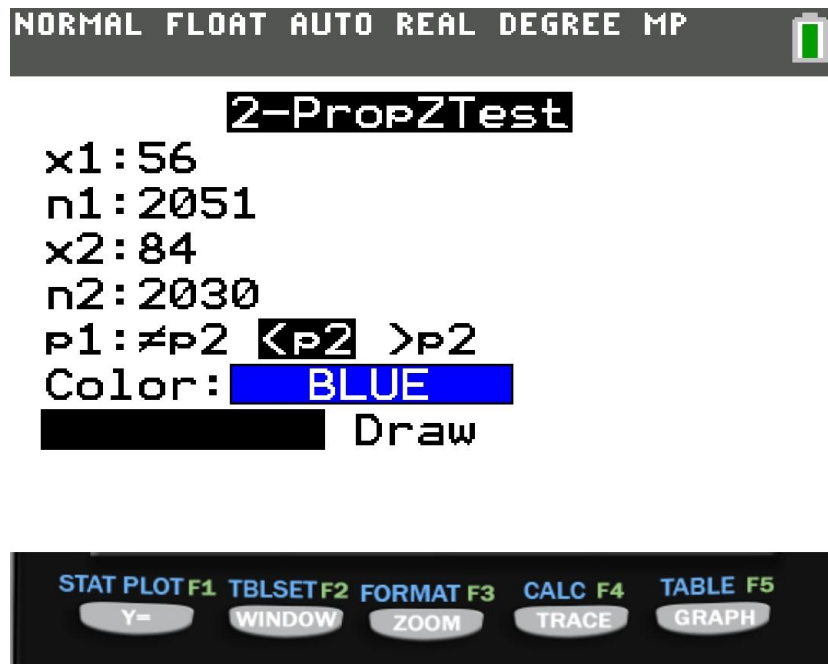
# Calculator steps

Step 1: open up the desired test. Press  
stat → scroll to test → select 2-propZTest



# Calculator steps

Step 2: enter the count values for each proportion. Note that it does require counts and not sample proportions. Choose the appropriate hypothesis. Then calculate.

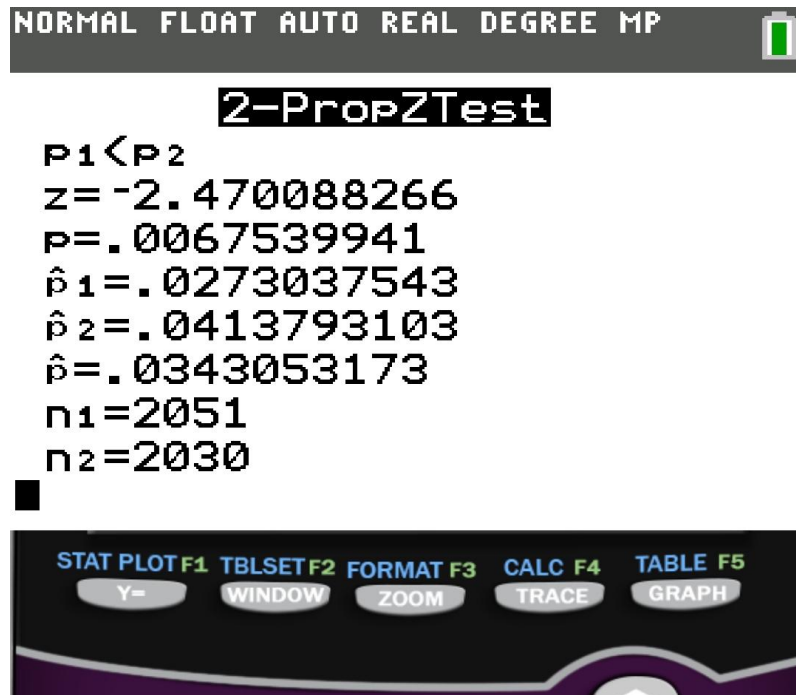




# Calculator steps

This provides the results of the test. Not the z-score of -2.47 and p-value of 0.00675... pretty close to what we got by hand, the difference is just rounding.

Now write down that you used a TI-84 calculator, that you used the 2-PropZTest, and all of the results that it lists out on the screen. The AP test is looking for all that info, plus a good statistician communicates their methods.



# Conclude

Not that the  $p\text{-value} < \alpha$  so we can reject the null hypothesis.

There is evidence at the  $\alpha = 0.05$  level to reject the null hypothesis ( $p\text{-value} = 0.0068$ ) that the men with and without the medication are having the same level of heart attacks. This suggests that the medication might be reducing the proportion of men who have heart attacks.

- Critical thinking question: what type of study is described in the scenario? What kinds of inference can we make on this type of study? Review chapter 4 if you are unsure.

# What about an interval

- We can also calculate an estimate of how large we think this difference really is. This also requires the 4 step process, but it is mostly the same.
- We are going to cover the parts of each step that are different!

# State

## Similarities:

- We define the two parameters here as well.

## Differences:

- Stating problem: We want to estimate the true mean difference between the proportion of men having heart attacks on the medication and on the placebo.
- Omit the hypotheses. Instead define the difference:

$P_1 - P_2$  : the true difference in the proportion of men having heart attacks on medicine vs on placebo.

# Plan

## Similarities:

- The assumptions are the same!

## Differences:

- Stating the test: We are going to conduct a 2-proportion z interval with 95% confidence

# Do

## Similarities:

- The sample sizes, proportions, and mean difference are the same.

## Differences

- We do not pool the probabilities for intervals because we are not assuming a null.

$$\sigma_{p_1-p_2} = \sqrt{\frac{P_c(1-P_c)}{n_1} + \frac{P_c(1-P_c)}{n_2}} = \sqrt{\frac{0.0273(1-0.0273)}{2051} + \frac{0.0414(1-0.0414)}{2030}} \approx 0.00570$$

- We use the confidence interval equation instead of the z-score equation:

$$C.I. = \mu_{p_1-p_2} \pm z^*(S.E.) = -0.0141 \pm 1.96 (0.00570) = (-0.0252, -0.0029)$$

# Do

A quick note about the pooling. In this case the difference between the pooled and non-pooled standard error was negligible. That will not always be case! The values will usually be close but not exact. The question of pooling comes down to the assumptions we are making. In a test, we are always assuming a null value. In a confidence interval, we are estimating a value. The later does not assume a null hypothesis, so we do not need to pool.

# Calculator steps!

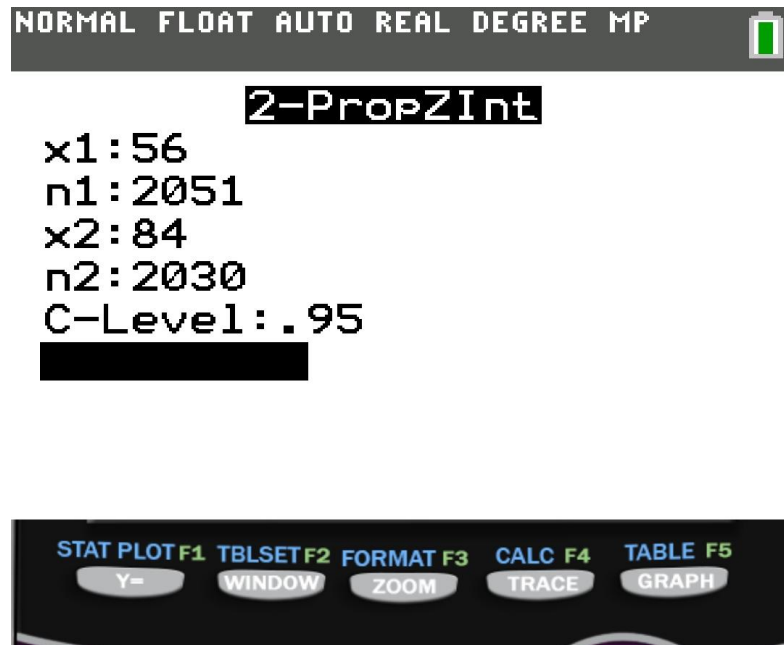
Step 1: go into the stat menu, scroll to tests, and choose 2-PropZInt.





# Calculator steps!

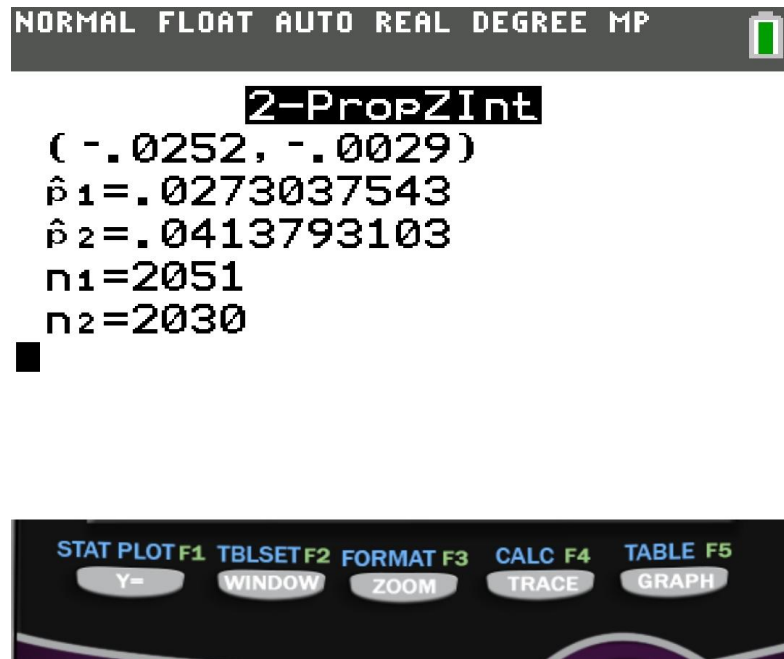
Step 2: Enter in the count and sample size data, then enter in chosen confidence level.  
Press calculate.



# Calculator steps!

Step 3: state the calculator and function used to calculate the interval, and copy down all info given by the calculator.

Again, good communication of your methods is essential in statistics.



# Conclude

We are 95% confident that the true mean difference in the proportion of men having heart attacks on the medication vs on the placebo is captured by the interval  $-0.0252$  to  $-0.0029$ . Therefore, we have evidence that suggests the medication is lowering the risk of heart attack. (why can I make a causal statement here?)

# Extra Practice

Reading: pg 604-619

HW: 1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23